Assume that you are thirsty. You are offered two alternatives:

a.- you can get glass of water now
b.- you can get the same glass of water in an hour

Would you value the two alternatives equally?

No. Other things being equal, you would prefer to get the glass of water now rather than having to wait an hour to drink it! The idea behind the concept of the time value of money is very similar. People prefer to receive €100 now rather than having to wait to get the same €100 at some time in the future. Why is this so?

1.- THE FUTURE VALUE OF MONEY

Suppose that you need €100 to pay for something right now. Then it is obvious why you prefer to receive €100 now rather than having to wait for it. However, suppose that you need to pay €100 in a year’s time. You would still prefer to receive the cash right now, even if you do not need it immediately. If you receive €100 now and do not need it for a year, you can always put this money into a savings account. Let us assume that this saving account is characterized by a 2% (0.02) annual interest rate. This means that for each €100 deposited in the account, you will receive an extra €2 after a year. So after one year the original €100 will become €102 or, in other words, you will be €2 richer. So when the time comes and you have to pay €100, you will be left with €2 to use however you like. This is the reason why even if you do not need the money right now, you should still prefer to receive €100 now rather than €100 in a year’s time.

Going back to the example of the savings account, €102 is the future value, in a year’s time of €100 now at a 2% annual interest rate. Let us call the annual interest rate i and the money received now PV.

The future value (FV) of PV in a year’s time at the annual rate i is equal to:

\[ FV = PV + PV \times i = PV(1+i) \]

In our example: \[ €102 = €100 + (0.02 \times €100) = €100(1+0.02) \]

Suppose now that after a year has passed, you still do not need the money and you leave it in the savings account for another year. How much money will you have at the end of the second year? We can apply the formula of the future value again, but this time we start from 102€. So you will have:

\[ €102 + (0.02 \times €102) = €102 (1+ 0.02) = €104.04 \]
This could be rather surprising. After leaving €100 in a savings account for two years that pays an annual interest rate of 2%, you may expect to have €104 at the end of the second year. Instead you have €104.04. Why? Because you start the second year with €102 in the account and therefore during the second year you earn a 2% interest rate on €102, instead, of only on €100. 2% of €2 is exactly €0.04, which is the additional amount of money that you have in your account at the end of the second year. This phenomenon is called the compounding of the interest rate. It means that every period the interest is calculated on the total amount obtained so far, and therefore also on the interest received in previous periods.

So after two periods (years) the future value (FV) of a sum of money (PV) at an interest rate i is equal to:

\[ FV = PV + (PVi) + (PV+ (PVi))i = PV (1+i)^2 \]

More generally, the future value (FV) of a sum of money (PV) after t periods, when the per period interest rate is i, is equal to:

\[ FV = PV(1+i)^t \]

### 2. The present value of money

Suppose now that you are promised to be paid €102 in a year's time. How much is this promise worth to you now? In order to answer this question you should ask yourself, "If I were given the chance to get the money right now, how much money would I ask for in order to be indifferent between the two alternatives?"

You already know the answer, €100! In the previous section we worked out that €100 now (assuming a 2% annual interest rate) has a future value of €102. So €100 is the present value of €102 in a year's time if the annual interest rate is 2%.

More generally, the calculation of the present value of a sum of money to be received in the future is exactly the inverse of the calculation of the future value of a sum of money to be received now. Therefore the present value (PV) of a sum of money (FV) to be received after t periods when the per period interest rate is i, is equal to:

\[ PV = \frac{FV}{(1+i)^t} = \frac{1}{(1+i)^t} FV \]

As you can see, this formula is simply the inverse of the formula we used to calculate the future value (FV).

\[ \frac{1}{(1+i)^t} \]

is called the discount factor because it is the number that transforms 1 unit of money after t periods into units of money, at the present moment, when the per period interest rate is equal to i. This number is smaller than one and becomes smaller, as the per period interest rate increases. This is logical. The higher the per-period interest rate, the more potential interest income we lose if we wait to receive a certain amount of money.

The idea of discounting is of crucial importance in business. Why do you think shops are ready to offer discounts for immediate cash payments? Because otherwise they have to wait to be paid and they loose potential interest income.
3.- THE PRESENT VALUE OF AN ANNUITY

An annuity is a constant payment that occurs every period for a fixed number of periods. For example if you are told that you will receive an annual annuity of €100 for five years, it means that you will receive €100 at the end of each of the following five years. What is the present value of such an annuity?

You can answer this question by applying the present value formula described in the previous section:

\[
PV = \frac{100}{1+i} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \frac{100}{(1+i)^4} + \frac{100}{(1+i)^5}
\]

More generally the present value (PV) of an annuity (A) for n periods, when the per-period interest rate is equal to i, is:

\[
PV = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots + \frac{A}{(1+i)^n} = A \left( \frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n} \right)
\]

As you can see, the value of the annuity (A), is multiplied by a special sum of discount factors. It can be shown that:

\[
\left( \frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n} \right) = \frac{1 - \left( \frac{1}{1+i} \right)^n}{i}
\]

The present value of an annuity can be also calculated by using the shortened formula

\[
PV = A \left( \frac{1 - \left( \frac{1}{1+i} \right)^n}{i} \right)
\]

4.- ANNUAL AND SEMIANNUAL INTEREST RATES

When the interest rate is annual, it means that the interest income generated by a certain sum of money is calculated once every 12 months.

When the interest rate is semi-annual, it means that the interest income generated by a certain sum of money is paid once every six months.

Assume that the annual interest rate is i. The equivalent semi-annual interest rate r is:

\[
r = \frac{i}{2}
\]