Bonds are financial documents that companies, governments and other entities use to raise cash in order to finance their businesses.

There are two possible ways for a company to finance its business:

- **Equity financing:** In this case the investor provides cash (or other assets) to the company in exchange for a portion of the ownership of the company, i.e., the investor becomes one of the owners of the company.

- **Debt financing:** In this case the investor provides cash to the company in exchange for a promise made by the company to pay back cash to the investor at a regular date in the future. Here, the investor does not become one of the owners of the company.

Bonds are financial instruments that companies use to raise debt financing.

### 1.- MAIN FEATURES OF BONDS

What is a bond? A bond is a financial document that specifies the terms and the timing of a series of cash flows that the company offers to pay to potential investors.

There are four essential elements specified in these types of document:

1.- The duration of the commitment of the company to pay cash flows to the investor. This period of time, usually measured in years, is called the life or the duration of the bond.

2.- The amount of money that the company will pay the investor at the end of the life of the bond. This is usually called the nominal value of the bond.

3.- The amount of money that the company promises to pay the investor at regular time intervals during the life of the bond. This amount of money is usually determined as a percentage of the nominal value of the bond. This percentage is called the coupon (nominal) rate of the bond, whereas the amount of money paid regularly by the company to the investor is called the coupon payment.

4.- The frequency at which the company promises to pay the coupon payment to the investor. Usually it is either annual or semiannual.
1.1- EXAMPLE (PART 1)

Life of the bond: 10 years  
Nominal value: €100,000  
Coupon rate: 10%  
Coupon payment: 0.10 x 100,000 = €10,000  
Frequency: annual

In this case, the bond states that the company will pay the investor €10,000 every year for the next ten years. Moreover, at the end of the tenth year, the company will also pay a lump sum of €100,000.

By playing with the nominal value, coupon rate and the frequency, a company can create a great variety of different cash flows. For example, there is also a financial instrument called zero-coupon bonds. These bonds simply represent a promise to pay the nominal value at the end of the life of the bond. In this case, the cash flow associated with the bond is simply a lump sum payment at the end of the life of the bond.

However, a bond is generally a combination of an annuity (fixed regular payments) during the life of the bond and a lump sum payment at the end of the life of the bond.

2.- THE MARKET VALUE OF A BOND

How much is a bond worth to an investor?

We already know the answer to this question. The value of the bond for an investor is the present value of the cash flows associated with the bond. The only additional information we need in order to calculate this present value is the interest rate at which we have to discount the future cash flows associated with the bond. This rate is usually called the market interest rate.

We can use the same coupon rate of the bond. In this case, the present value of the cash flows associated with the bond will be exactly equal to the nominal value of the bond. This is not abnormal. If the yearly cost of having to wait until the end of the life of the bond to get the nominal value of the bond is equal to the rate used to calculate the yearly coupon payments of the bond, then the cost to wait to get paid is exactly compensated by the periodic income generated by the bond. In the end, the bond is worth exactly its nominal value.
2.1- EXAMPLE (PART 2)

Let us go back to the bond described before.

If the discount rate is equal to the coupon rate, i.e., it is equal to 10% per year, then the present value of the bond can be calculated as follows;

\[ PV = \frac{10000}{(1 + 0.1)} + \frac{10000}{(1 + 0.1)^2} + \frac{10000}{(1 + 0.1)^3} + \frac{10000}{(1 + 0.1)^4} + \frac{10000}{(1 + 0.1)^5} + \frac{10000}{(1 + 0.1)^6} + \frac{10000}{(1 + 0.1)^7} + \]

\[ + \frac{10000}{(1 + 0.1)^8} + \frac{10000}{(1 + 0.1)^9} + \frac{10000}{(1 + 0.1)^{10}} + \frac{100000}{(1 + 0.1)^{10}} = 100000 \left(1 - \frac{1}{(1 + 0.1)^{10}}\right) / 0.1 \]

So if the market rate is equal to the coupon rate, we say that the bond is issued at par because its market (present) value is equal to its nominal value. In this situation, an investor will pay the nominal value to the company in order to acquire the bond. Although under normal circumstances, the market rate does not coincide with the coupon rate of the bond.

The rate at which an investor in the market will discount the future “promises to pay” of a certain company will depend on many factors. Generally we can say that it will be equal to a certain “risk free market rate” plus a “risk premium” that is characteristic of the company. The “risk free market rate” is an economy wide rate at which money can be invested at no risk. The “risk premium” is an additional percentage that is added to the “risk free rate” in order to take into account the risks involved in lending money to a specific company. So if the risk free rate is 4% and the risk premium associated to company A is 2%, the rate at which the promises to pay money in the future made by company A are discounted by the market is 6% (4%+2%). To discuss at length the determination of the market interest rate for a certain bond goes beyond the scope of this short note and is a topic for a corporate finance course. For our accounting purposes, we will take the market interest rate as an exogenous given number determined in the market for corporate bonds.

If the market rate is higher than the coupon rate, then the present value of the cash flows associated with the bond will be less than the nominal value.
2.2- EXAMPLE (PART 3)

Let us assume now that the market rate is 11%. The present value of the cash flows associated with our 10 year, 10% annual coupon rate, €100,000 nominal value bond is:

\[ PV = \frac{10000}{(1 + 0.11)} + \frac{10000}{(1 + 0.11)^2} + \frac{10000}{(1 + 0.11)^3} + \frac{10000}{(1 + 0.11)^4} + \frac{10000}{(1 + 0.11)^5} + \frac{10000}{(1 + 0.11)^6} + \frac{10000}{(1 + 0.11)^7} + \]
\[ + \frac{10000}{(1 + 0.11)^8} + \frac{10000}{(1 + 0.11)^9} + \frac{10000}{(1 + 0.11)^{10}} = \frac{1}{(1 + 0.11)^{10}} \cdot (0.1 \times 100000) + \frac{100000}{(1 + 0.11)^{10}} = \]
\[ = 100000 \left( \frac{0.1}{0.11} \left( 1 - \frac{1}{(1 + 0.11)^{10}} \right) + \frac{1}{(1 + 0.11)^{10}} \right) = 94111 < 100000 \]

If the market rate is greater than the coupon rate, we say that the bond is issued at a discount because its market (present) value is less than its nominal value. In this situation an investor will pay less than the nominal value to the company in order to acquire the bond. If the market rate is lower than the coupon rate, then the present value of the cash flows associated with the bond will be greater than the nominal value.

2.3- EXAMPLE (PART 4)

Let us assume now that the market rate is 9%. The present value of the cash flows associated with our 10 year, 10% annual coupon rate, €100,000 nominal value bond is:

\[ PV = \frac{10000}{(1 + 0.9)} + \frac{10000}{(1 + 0.9)^2} + \frac{10000}{(1 + 0.9)^3} + \frac{10000}{(1 + 0.9)^4} + \frac{10000}{(1 + 0.9)^5} + \frac{10000}{(1 + 0.9)^6} + \frac{10000}{(1 + 0.9)^7} + \]
\[ + \frac{10000}{(1 + 0.9)^8} + \frac{10000}{(1 + 0.9)^9} + \frac{10000}{(1 + 0.9)^{10}} = \frac{1}{(1 + 0.9)^{10}} \cdot (0.1 \times 100000) + \frac{100000}{(1 + 0.9)^{10}} = \]
\[ = 100000 \left( \frac{0.1}{0.9} \left( 1 - \frac{1}{(1 + 0.9)^{10}} \right) + \frac{1}{(1 + 0.9)^{10}} \right) = 106418 > 100000 \]

If the market rate is lower than the coupon rate, we say that the bond is issued at a premium because its market (present) value is greater than its nominal value. In this situation an investor will pay more than the nominal value to the company in order to acquire the bond.

3.- THE ECONOMICS OF A BOND

We should understand by now, the economic logic behind the cash flows associated with a bond. An investor that has some funds to invest buys the bond at its market (present) value. This is associated with a cash flow from the investor to the company. Afterwards, the company should fulfill the promises written on the bonds by paying the regular coupon payments to the investor. These payments are associated with regular cash flows from the company to the investor. Finally, at the end of the life of the bond, the company pays the investor the nominal value of the bond. This payment is associated with a cash flow from the company to the investor.

If the bond is issued at discount, the financial conditions (coupon rate) attached to the cash flows paid by the company to the investor are more favorable than those offered by the investor (market
rate). Therefore, in order to sell the bond, the company has to pay an additional financial cost: the difference between the nominal value of the bond and its market value. This additional financial cost is the discount.

If the bond is issued at premium, the financial conditions (coupon rate) attached to the cash flows paid by the company to the investor are less favourable than those offered by the investor (market rate). So, the company can sell the bond at an additional financial gain: the difference between the market rate of the bond and its nominal value. This additional financial gain is the premium.

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